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Application of the Green functions to the problem of the thermally induced vibration of a beam

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Abstract

This paper considers the problem of the transverse vibrations of a beam induced by a mobile heat source. The formulation of the problem is based on the differential equations of heat conduction and transverse vibrations of the beam, which are complemented by suitable initial and boundary conditions. The effect of internal and external damping on the vibrations of the beam is considered. The solution to the problem in analytical form is obtained by using the properties of the Green functions. A time partitioning method has been used to avoid the difficulties associated with the slow convergence of the series occurring in the solution to the heat conduction problem. The numerical results of the thermally induced vibration of the beam are presented.

1. Introduction

Changes in the temperature of a beam produce thermal stresses, which cause displacements of the beam. Cyclic changes in the temperature of the beam induce transverse vibrations. The problem of thermally induced vibrations of beams has been considered by Yu [1], Boley [2,3], Manolis and Beskos [4] and Kidawa-Kukla [5].

Yu [1] extensively explored the problem of thermal flutter of a spacecraft boom. In that paper, the author studied the effect of viscoelastic damping and a viscous fluid damper on the stability of the boom's motion. The vibrations of a simply supported, rectangular beam, subjected to a suddenly applied heat input distributed along its span, were analyzed by Boley [3]. Manolis and Beskos [4] examined thermally induced vibrations of structures composed of beams, exposed to rapid surface heating. The authors have also studied the effects of damping and of axial loads on the structural response. Kidawa-Kukla [5] presented a solution to the problem of thermally induced vibration of a simply supported beam. In that paper the temperature of the beam changes

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as a result of heating by a laser beam. The analytical form of the solution was obtained by using the properties of the Green functions. The application of the Green function method for heat conduction problems is presented by Beck et al. [6]. A time partitioning method is introduced to improve the evaluation of the Green function solutions. The thermally induced displacement and stresses of a rod are investigated by Al-Huniti et al. [7]. The problem is solved by using the Laplace transformation technique.

In this paper the problem of transverse vibrations of a beam induced by a mobile heat source is considered. The effects of internal as well as external damping on the vibrations have been taken into account. The problem is solved by using the properties of the Green functions. The temperature distribution in the beam is obtained by applying time partitioning to the Green function method.

2. Problem formulation

Consider a uniform beam of length L, with a rectangular cross-section of width b and height h as shown in Fig. 1. The differential equation of thermally induced lateral vibration of the beam with internal and external damping may be written in the form

$$EI\left[\frac{\partial^4 v}{\partial x^4}+f\frac{\partial^5 v}{\partial x^4 \partial t}\right]+c_d\frac{\partial v}{\partial t}+\rho\frac{\partial^2 v}{\partial t^2}=Q(x,t), \tag{1}$$

where EI is the bending rigidity, ρ is the mass per unit length of the beam, v is the lateral beam deflection, f is the internal damping coefficient of the beam material, c_d is the external damping coefficient, x is the distance along the length of the beam and t denotes time. The transverse vibrations of the beam are induced by the thermal load

$$Q(x,t)=\alpha bE\int_0^h\left(y-\frac{h}{2}\right)\frac{\partial^2 T(x,y,t)}{\partial x^2}\,\mathrm{d}y, \tag{2}$$

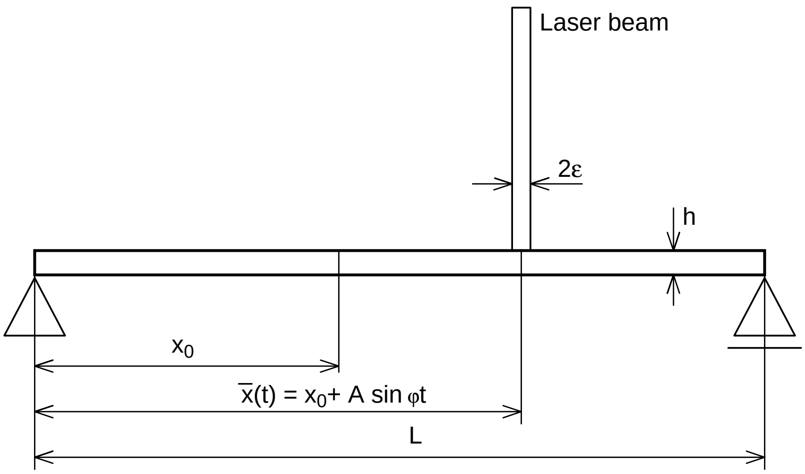

Fig. 1. The beam model considered.

where α is the coefficient of thermal expansion, E is Young's modulus, $T(x, y, t)$ is the temperature distribution in the beam. Eq. (1) is complemented by the zero-value initial conditions

$$v(x,0) = \frac{\partial v}{\partial t}(x,0) = 0 \tag{3}$$

and the boundary conditions corresponding to the simply supported beam

$$v(0,t) = v(L,t) = 0, \quad \frac{\partial^2 v}{\partial x^2}(0,t) = \frac{\partial^2 v}{\partial x^2}(L,t) = 0. \tag{4}$$

The temperature distribution in the beam is described by the differential equation of heat conduction, which takes into account the activity of the heat source

$$\nabla^2 T + \frac{1}{\lambda} g(x,y,t) = \frac{1}{\kappa}\frac{\partial T}{\partial t}, \tag{5}$$

where $\nabla^2 \equiv (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ is the Laplace's operator, λ is the thermal conductivity, κ is the thermal diffusivity. The term $g(x, y, t)$ represents the volume energy generation

$$g(x,y,t) = \begin{cases} \dfrac{\theta}{2\varepsilon}\delta(y) & \text{for } \bar{x}(t) - \varepsilon < x < \bar{x}(t) + \varepsilon, \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

where θ characterizes the stream of heat, $\delta(\cdot)$ is the Dirac delta function and 2ε is the width of the element on the surface of the beam heated by the heat source.

The heat source moves harmonically over a fixed point on the beam and always remains within the length of the beam. The function $\bar{x}(t)$ describes the movement of the heat source

$$\bar{x}(t) = x_0 + A \sin \varphi t, \tag{7}$$

where $A + \varepsilon < x_0 < L - A - \varepsilon$.

The temperature distribution in the beam is obtained as a solution to Eq. (5) with the following initial and boundary conditions:

$$T(x,y,0) = T_i, \tag{8}$$

$$T(0,y,t) = T(L,y,t) = 0, \tag{9}$$

$$\lambda \frac{\partial T}{\partial y}(x,0,t) = -\bar{\alpha}_0 [T_0 - T(x,0,t)], \tag{10}$$

$$\lambda \frac{\partial T}{\partial y}(x,h,t) = \bar{\alpha}_1 [T_1 - T(x,h,t)], \tag{11}$$

where $\bar{\alpha}_0$, $\bar{\alpha}_1$ are the heat transfer coefficients, T_i is the initial temperature, T_0 and T_1 are the temperatures of the surrounding medium.

3. Solution to the problem

3.1. Temperature distribution in the beam

The solution to the problem is obtained by using the properties of the Green functions. The Green function of the heat conduction problem G_T describes the temperature distribution induced by the temporary, local energy impulse. The function is a solution to the differential equation

$$\kappa\nabla^2 G_T + \frac{\partial G_T}{\partial \tau} = -\delta(x-\xi)\delta(y-\eta)\delta(t-\tau). \tag{12}$$

It is assumed that the Green function satisfies the same boundary conditions as the temperature function (Eqs. (9)–(11)).

The temperature distribution $T(x, y, t)$ is expressed by the Green function G_T as follows:

$$\begin{aligned} T(x,y,t) =& \frac{\theta\kappa}{2\varepsilon\lambda}\int_0^t \int_{\bar{x}(t)-\varepsilon}^{\bar{x}(t)+\varepsilon} G_T(x,y,t;\xi,0,\tau)\,\mathrm{d}\xi\,\mathrm{d}\tau + T_i\int_{\xi=0}^{\xi=L}\mathrm{d}\xi\int_{\eta=0}^{\eta=L} G_T(x,y,t;\xi,\eta,0)\,\mathrm{d}\eta \\ &+ \kappa\int_{\tau=0}^{\tau=t+\varepsilon}\mathrm{d}\tau\int_{\xi=0}^{\xi=L}\mathrm{d}\xi\left[\mu_0 T_0 G_T(x,y,t,\xi,0,\tau) + \mu_1 T_1 G_T(x,y,t,\xi,h,\tau)\right], \end{aligned} \tag{13}$$

where $\mu_0 = \bar{\alpha}_0/\lambda$, $\mu_1 = \bar{\alpha}_1/\lambda$.

The integrals in Eq. (13) are then expressed by infinite series. To avoid the difficulties associated with the slow convergence of these series the time partitioning method has been used [6]. In this method, the time interval (0, t) is divided into two intervals: $(0, t-\Delta t)$ and $(t-\Delta t, t)$, where $0<\Delta t<t$. The Green functions of the heat conduction problem for a small-time G_T^S and for a large-time G_T^L are distinguished. For clarity, the method will be presented below using as an example the first integral occurring on the right-hand of Eq. (13).

The function T may be written in the form

$$T(x,y,t) = T_S(x,y,t) + T_L(x,y,t), \tag{14}$$

where

$$T_S(x,y,t) = \frac{\theta\kappa}{2\varepsilon\lambda}\int_{t-\Delta t}^{t}\int_{\bar{x}(\tau)-\varepsilon}^{\bar{x}(\tau)+\varepsilon} G_T^s(x,y,\xi,0,t-\tau)\,\mathrm{d}\xi\,\mathrm{d}\tau, \tag{15}$$

$$T_L(x,y,t) = \frac{\theta\kappa}{2\varepsilon\lambda}\int_{0}^{t-\Delta t}\int_{\bar{x}(\tau)-\varepsilon}^{\bar{x}(\tau)+\varepsilon} G_T^L(x,y,\xi,0,t-\tau)\,\mathrm{d}\xi\,\mathrm{d}\tau. \tag{16}$$

The function G_T^L reads [8]

$$G_T^L(x,y,\xi,\eta,t-\tau) = \frac{4}{hL}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{q_n^2}\sin\alpha_m x\sin\alpha_m\xi\,\Psi_n(y)\Psi_n(\eta)\exp(-\gamma_{mn}(t-\tau)), \tag{17}$$

where

$$\alpha_m = m\pi/L, \quad \gamma_{mn} = \kappa[\alpha_m^2+\beta_n^2], \quad q_n^2 = \mu_0^2+\beta_n^2+\frac{(\mu_0+\mu_1)}{h}\frac{(\mu_0\mu_1+\beta_n^2)}{(\mu_1^2+\beta_n^2)},$$

$$\Psi_n(y) = \beta_n\cos\beta_n y + \mu_0\sin\beta_n y \quad \text{for } m,n = 1,2,\ldots$$

and β_n are roots of the equation

$$\mu_0^2 - \beta_n^2 + (\mu_0 + \mu_1)\beta_n \operatorname{ctg} \beta_n h = 0. \tag{18}$$

Substituting the Green function (17) into Eq. (16) gives

$$T_L(x, y, t) = \frac{8\theta\kappa}{hL\lambda} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn}^T(t) \frac{\beta_n}{q_n^2} \frac{\sin \alpha_m \varepsilon}{\alpha_m \varepsilon} \Psi_n(y) \sin \alpha_m x, \tag{19}$$

where

$$D_{mn}^T = \frac{1}{2} \int_0^{t-\Delta t} \exp[-\gamma_{mn}(t-\tau)] \sin[\alpha_m(x_0 + A \sin \varphi\tau)] \, \mathrm{d}\tau. \tag{20}$$

Integral (20) is evaluated by using the following relationships [9]:

$$\cos(r \sin u) = 2 \sum_{i=0}^{\infty} \chi_i \mathrm{J}_{2i}(r) \cos 2iu, \tag{21}$$

$$\sin(r \sin u) = 2 \sum_{i=0}^{\infty} \mathrm{J}_{2i+1}(r) \sin(2i+1)u, \tag{22}$$

where $\chi_0 = 0.5$, $\chi_i = 1$ for $i = 1, 2, \ldots$ and $\mathrm{J}_v(\cdot)$ denotes the Bessel function of the first kind of order v. Finally, Eq. (20) can be written in the form

$$\begin{aligned} D_{mn}^T(t) = {} & \sin \alpha_m x_0 \sum_{i=0}^{\infty} \chi_i \mathrm{J}_{2i}(A\alpha_m) \frac{1}{\gamma_{mn}^2 + 4i^2\varphi^2} U_{imn}^T(t) \\ & + \cos \alpha_m x_0 \sum_{i=0}^{\infty} \mathrm{J}_{2i+1}(A\alpha_m) \frac{1}{\gamma_{mn}^2 + (2i+1)^2\varphi^2} W_{imn}^T(t), \end{aligned} \tag{23}$$

where

$$U_{imn}^T(t) = [2i\varphi \sin 2i(t-\Delta t)\varphi + \gamma_{mn} \cos 2i(t-\Delta t)\varphi] \mathrm{e}^{-\gamma_{mn}\Delta t} - \gamma_{mn} \mathrm{e}^{-\gamma_{mn} t}, \tag{24}$$

$$\begin{aligned} W_{imn}^T(t) = {} & [\gamma_{mn} \sin(2i+1)(t-\Delta t)\varphi - (2i+1)\varphi \cos(2i+1)(t-\Delta t)\varphi] \mathrm{e}^{-\gamma_{mn}\Delta t} \\ & + (2i+1)\varphi \mathrm{e}^{-\gamma_{mn} t}. \end{aligned} \tag{25}$$

The Green function G_T^S for the small time may be written in the form [6]

$$G_T^s(x, y, \xi, \eta, t-\tau) \approx G_{11}^s(x, \xi, t-\tau) G_{33}^s(y, \eta, t-\tau), \tag{26}$$

where

$$G_{11}^s(x, \xi, t-\tau) \approx \frac{1}{2\sqrt{\pi\kappa(t-\tau)}} \left\{ \exp\left[-\frac{(x-\xi)^2}{4\kappa(t-\tau)}\right] - \exp\left[-\frac{(x+\xi)^2}{4\kappa(t-\tau)}\right] - \exp\left[-\frac{(2L-x-\xi)^2}{4\kappa(t-\tau)}\right] \right\} \tag{27}$$

and

$$G_{33}^s(y,\eta,t-\tau) \approx \frac{1}{2\sqrt{\pi\kappa(t-\tau)}}\left\{\exp\left[-\frac{(y-\eta)^2}{4\kappa(t-\tau)}\right]+\exp\left[-\frac{(y+\eta)^2}{4\kappa(t-\tau)}\right]+\exp\left[-\frac{(2h-y-\eta)^2}{4\kappa(t-\tau)}\right]\right\}$$
$$-\frac{\bar{\alpha}_0}{\lambda}\exp\left[\frac{(y+\eta)\bar{\alpha}_0}{\lambda}+\frac{\bar{\alpha}_0^2\kappa(t-\tau)}{\lambda^2}\right]\mathrm{erfc}\left\{\frac{y+\eta}{2\sqrt{\kappa(t-\tau)}}+\frac{\bar{\alpha}_0}{\lambda}\sqrt{\kappa(t-\tau)}\right\}$$
$$-\frac{\bar{\alpha}_1}{\lambda}\exp\left[\frac{(2h-y-\eta)\bar{\alpha}_1}{\lambda}+\frac{\bar{\alpha}_1^2\kappa(t-\tau)}{\lambda^2}\right]\mathrm{erfc}\left\{\frac{2h-y-\eta}{2\sqrt{\kappa(t-\tau)}}+\frac{\bar{\alpha}_1}{\lambda}\sqrt{\kappa(t-\tau)}\right\}. \quad (28)$$

Substituting the Green functions (27–28) into Eqs. (26) and (15), the function $T_S(x, y, t)$ is expressed as

$$T_S(x,y,t)=\frac{\theta\kappa}{2\varepsilon\lambda}\int_{t-\Delta t}^{t} G_{33}^s(y,0,t-\tau)\,K(x,t,\tau)\,\mathrm{d}\tau, \quad (29)$$

where

$$K(x,t,\tau)=\int_{\bar{x}(\tau)-\varepsilon}^{\bar{x}(\tau)+\varepsilon} G_{11}^s(x,\xi,t-\tau)\,\mathrm{d}\xi=\frac{1}{2}\left\{\mathrm{erf}\left[\frac{x-\bar{x}(\tau)+\varepsilon}{2\sqrt{\kappa(t-\tau)}}\right]-\mathrm{erf}\left[\frac{x-\bar{x}(\tau)-\varepsilon}{2\sqrt{\kappa(t-\tau)}}\right]\right.$$
$$-\mathrm{erf}\left[\frac{x+\bar{x}(\tau)+\varepsilon}{2\sqrt{\kappa(t-\tau)}}\right]+\mathrm{erf}\left[\frac{x+\bar{x}(\tau)-\varepsilon}{2\sqrt{\kappa(t-\tau)}}\right]$$
$$\left.-\mathrm{erf}\left[\frac{2L-x-\bar{x}(\tau)+\varepsilon}{2\sqrt{\kappa(t-\tau)}}\right]+\mathrm{erf}\left[\frac{2L-x-\bar{x}(\tau)-\varepsilon}{2\sqrt{\kappa(t-\tau)}}\right]\right\}. \quad (30)$$

The integral in Eq. (29) is then calculated numerically.

3.2. Transverse vibrations of the beam

The problem of vibrations of the beam is solved by using the Green function G_V, which is a solution to the equation

$$EI\left[\frac{\partial^4 G_v}{\partial x^4}+f\frac{\partial^5 G_v}{\partial x^4\partial t}\right]+c_d\frac{\partial G_v}{\partial t}+\rho\frac{\partial^2 G_v}{\partial t^2}=\delta(x-\xi)\delta(t-\tau) \quad (31)$$

and satisfies initial and boundary conditions analogous to the displacement function (Eqs. (3)–(4)). If function G_V is known, then function $v(x, t)$ may be written in the form

$$v(x,t)=\int_0^t\int_0^L Q(\xi,\tau)G_v(x,\xi,t-\tau)\,\mathrm{d}\xi\,\mathrm{d}\tau. \quad (32)$$

Function $Q(\xi,\tau)$ is found by substituting the function given by Eq. (13) into Eq. (2), where $G_T(x,y,t,\xi,\eta,\tau)=G_T^L(x,y,\xi,\eta,t-\tau)$.

The Green function G_V may be written in the form of a sine series [5]

$$G_v(x,t;\xi,\tau)=\frac{2}{\rho L}\sum_{m=1}^{\infty} w_m(t,\tau)\sin\frac{m\pi x}{L}\sin\frac{m\pi\xi}{L}, \quad (33)$$

where functions $w_m(t,\tau)$ are obtained as solutions to the equation

$$\ddot{w}_m + (\gamma + \omega_m^2 f)\dot{w}_m + \omega_m^2 w_m = \delta(t-\tau). \tag{34}$$

Finally, the Green function has the form [10]

$$G_v(x,\xi,t-\tau) = \frac{2}{\rho L}\mathrm{H}(t-\tau)\sum_{m=1}^{\infty} Y_m(t-\tau)\mathrm{e}^{-\zeta_m(t-\tau)}\sin\alpha_m x \sin\alpha_m \xi, \tag{35}$$

where

$$Y_m(u) = \begin{cases} \dfrac{1}{\Omega_m}\sin\Omega_m u, & \Delta < 0, \\ u, & \Delta = 0, \\ \dfrac{1}{\Omega_m}\sinh\bar{\Omega}_m u, & \Delta > 0 \end{cases} \tag{36}$$

and $\Delta = 4(\zeta_m^2 - \omega_m^2)$, $\Omega_m = \sqrt{\omega_m^2 - \zeta_m^2}$, $\bar{\Omega}_m = \sqrt{\zeta_m^2 - \omega_m^2}$, $\zeta_m = \frac{1}{2}(v + \omega_m^2 f)$, $\omega_m = \sqrt{(EI/\rho)}\alpha_m^2$, $v = c_d/\rho$, $\mathrm{H}(\cdot)$ denotes the Heaviside function.

On the basis of Eq. (32), the deflection of the beam is expressed as

$$v(x,t) = \frac{8Eh\alpha\kappa\theta}{\lambda\varepsilon\rho L}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_n D_{mn}^v(t)\frac{\alpha_m}{q_n^2}\sin\alpha_m\varepsilon\, sin\,\alpha_m x, \tag{37}$$

where

$$B_n = \left(\frac{\mu_0}{\beta_n} + \frac{\beta_n h}{2}\right)\sin\beta_n h - \left[\frac{\mu_0 h}{2} + 1 + \left(\frac{\mu_0 h}{2} - 1\right)\cos\beta_n h\right], \tag{38}$$

$$\begin{aligned} D_{mn}^v(t) &= \int_0^t D_{mn}^T(\tau) Y_m(t-\tau)\mathrm{e}^{-\zeta_m(t-\tau)}\,\mathrm{d}\tau \\ &= \sin\alpha_m x_0 \sum_{i=0}^{\infty}\chi_i J_{2i}(A\alpha_m)\frac{1}{\gamma_{mn}^2 + 4i^2\varphi^2}U_{imn}^v(t) \\ &\quad + \cos\alpha_m x_0 \sum_{i=0}^{\infty} J_{2i+1}(A\alpha_m)\frac{1}{\gamma_{mn}^2 + (2i+1)^2\varphi^2}W_{imn}^v(t) \end{aligned} \tag{39}$$

and

$$U_{imn}^v(t) = \int_0^t U_{imn}^T(\tau) Y_m(t-\tau)\mathrm{e}^{-\zeta_m(t-\tau)}\,\mathrm{d}\tau, \tag{40}$$

$$W_{imn}^v(t) = \int_0^t W_{imn}^T(\tau) Y_m(t-\tau)\mathrm{e}^{-\zeta_m(t-\tau)}\,\mathrm{d}\tau. \tag{41}$$

The integrals in Eqs. (40) and (41) are evaluated for three cases depending on the form of function Y_m given by Eq. (36). Functions $U_{imn}^v(t)$ and $W_{imn}^v(t)$ are given in the appendix.

In the case of the system without internal and external damping ($v = f = 0$), the integrals in Eqs. (40) and (41) assume a particular form. Namely, if $\zeta_m = 0$ and $\omega_m = 2i\varphi$ for any natural numbers 'i' and 'm', then the function $U^v_{imn}(t)$ is given as follows:

$$U^v_{imn}(t) = \int_0^t U^T_{imn}(\tau)\, Y_m(t-\tau)\,\mathrm{d}\tau = \frac{t}{2}\left[\cos\omega_m t - \frac{\gamma_{mn}}{\omega_m}\sin\omega_m t\right] + \frac{1}{\gamma^2_{mn}+\omega^2_m}\left[\gamma_{mn}\cos\omega_m t - \frac{\gamma^2_{mn}-\omega^2_m}{2\omega_m}\sin\omega_m t - \gamma_{mn}\exp(-\gamma_{mn}t)\right]. \quad (42)$$

Similarly, if $\zeta_m = 0$ and $\omega_m = (2i+1)\varphi$ for any numbers 'i' and 'm', then the function $W^v_{imn}(t)$ has the form

$$W^v_{imn}(t) = \int_0^t W^T_{imn}(\tau)\, Y_m(t-\tau)\,\mathrm{d}\tau = -\frac{t}{2\omega_m}\left[\frac{\gamma_{mn}}{\omega_m}\cos\omega_m t + \sin\omega_m t\right] + \frac{1}{\gamma^2_{mn}+\omega^2_m}\left[-\cos\omega_m t + \frac{\gamma_{mn}(\gamma^2_{mn}+3\omega^2_m)}{2\omega^3_m}\sin\omega_m t + \exp(-\gamma_{mn}t)\right]. \quad (43)$$

It follows that functions $U^v_{imn}(t)$ and $W^v_{imn}(t)$ include terms whose amplitude of oscillations increases with time t. This means that one member of series (37) has increasing oscillations. Hence, it results that in the case of the system without damping, the amplitude of beam vibration may increase with time t. It follows that resonance may occur in the system.

4. Numerical examples

A uniform, simply supported rectangular beam and a heat source which changes its position over point $x_0 = 0.5L$, according to Eq. (7) are considered. The numerical calculations are performed for both a steel and an aluminum beam. The following numerical data was assumed:

- for the steel beam: $E = 2\times10^{11}$ (N/m^2), $\rho = 3.12$ (kg/m), $c = 510$ (J/(kg K)), $\lambda = 51.4$ (W/(m K)), $\alpha = 1.25\times10^{-7}$ (1/K);
- for the aluminum beam: $E = 7\times10^9$ (N/m^2), $\rho = 1.08$ (kg/m), $c = 896$ (J/(kg K)), $\lambda = 229$ (W/(m K)), $\alpha = 2.37\times10^{-7}$ (1/K).

Moreover, for both beams it was assumed that: $L = 1.0$ (m), $b = h = 0.02$ (m), $A = 0.3$ (m), $\varepsilon = 0.001$ (m), $\bar{\alpha}_0 = \bar{\alpha}_1 = 100$ (W/(m^{2}K)), $\theta = 50{,}000$ (W/m), $T_0 = T_1 = 0°$C.

The numerical investigations of the effect of internal damping on the displacement of the beam for short- and long-time periods for both the steel and the aluminum beam (without external damping) were done by assuming $\varphi = 0.5$, $\omega_1 = 0.5(\pi/L)^2\sqrt{EI/\rho}$. In this case, condition $\omega_1 = 2\varphi$ is satisfied and for $f = 0$ Eqs. (37)–(39) and (42) are used to calculate the displacement. This means, that the amplitude of the vibrations increases with time (resonance case). In Figs. 2 and 3 the time histories of the displacements of the mid-point of the beam without internal damping (solid line) and with internal damping (dashed line) are shown. The numerical calculations demonstrate the inconspicuous influence of the internal damping on the vibration amplitude of the beam at the beginning of the process. The effect of the internal damping on the amplitude of the beam vibration is significant in the resonance case as a considerable decrease in the beam vibration amplitude is observed.

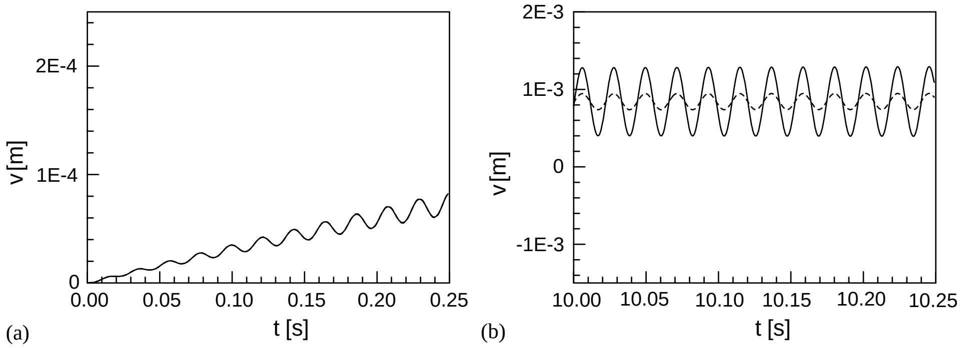

Fig. 2. Time histories of the displacements of the mid-point of the steel beam for $\varphi = 0.5\omega_1$ without damping (solid line) and with internal damping, $f = 10^{-5}$ (s) (dashed line); (a) displacement at the beginning of the process and (b) displacement after 10 s.

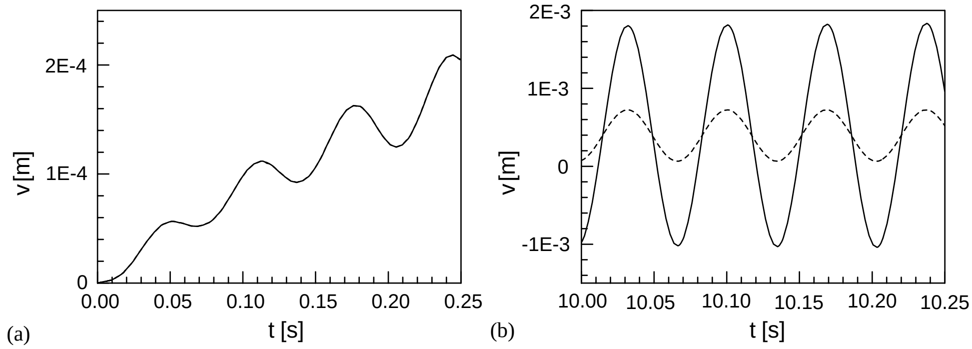

Fig. 3. Time histories of the displacements of the mid-point of the aluminum beam for $\varphi = 0.5\omega_1$ without damping (solid line) and with internal damping, $f = 10^{-4}$ (s) (dashed line); (a) displacement at the beginning of the process and (b) displacement after 10 s.

In Fig. 4, the displacement of the mid-point of the beam for $\varphi = 0.2\pi$ is presented (vibration of the beam without internal damping). In this case, the mobile heat source induced periodic vibrations of the beam. The calculations are performed for the external damping coefficient $v = 0$ (solid line) and $v = 5\times10^4$ (kg/(m s)) (dashed line). Fig. 4a shows the displacement for a short-time period and Fig. 4b shows the displacement for a long-time period. The vibration of the beam reaches a steady state soon after the beginning of the process. It is seen from the results that the external damping causes a significant decrease in the vibration amplitude of the aluminum beam, but only a slight decrease in the vibration amplitude of the steel beam.

5. Conclusions

The problem of thermally induced vibration of a beam has been considered. The temperature distribution and transverse vibration of the beam in analytical form are obtained by using the

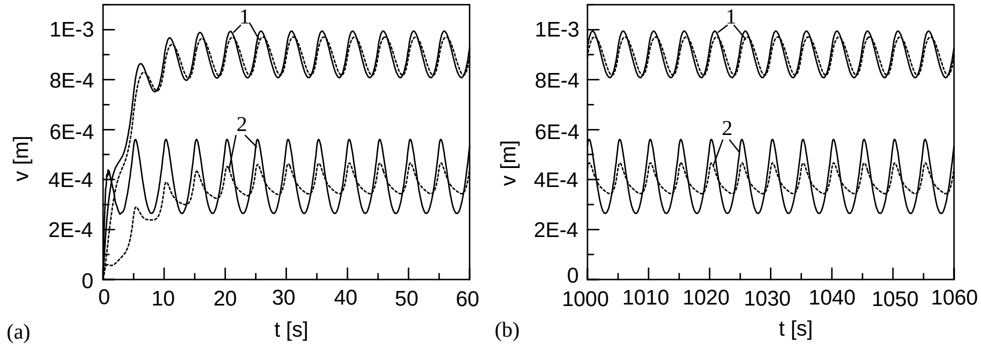

Fig. 4. Time histories of the displacements of the mid-point of the beam for $\varphi = 0.2\pi$ without damping (solid line) and with external damping $\nu = 5 \times 10^4$ (kg/ms) (dashed line), 1—the steel beam, 2—the aluminum beam; (a) displacement at the beginning of the process and (b) displacement after 1000 s.

properties of the Green functions. The solution to the problem is expressed by infinite series. The time partitioning method has been used to avoid the difficulties associated with the slow convergence of the series. The Green functions of the heat conduction problem are distinguished for a small time and a large time.

The presented formulation and solution of the vibration problem take into consideration the external damping and internal damping of the beam material. It follows from the solution of the problem that the amplitude of the beam vibration may increase with time. Such a situation happens in the case of vibration without damping, if one eigenfrequency of the beam is a multiple of the harmonic motion frequency of the heat source.

Numerical calculations performed for the steel and aluminum beams show that the vibration amplitude of the aluminum beam is significantly greater than the vibration amplitude of the steel beam. The effect of the internal damping on the vibration amplitude at the beginning of the process is inconspicuous. In the resonance case the internal damping of the material significantly decreases the amplitude of the steady state vibration. The external damping causes a significant decrease in the vibration amplitude of the aluminum beam, but only a slight decrease in the vibration amplitude of the steel beam.

Appendix

The integrals in Eqs. (40) and (41) may be evaluated for three different cases depending on the sign of expression $\Delta = 4(\zeta_m^2 - \omega_m^2)$. The results below concern these three cases except the one given in Section 3 of this paper.

Case 1: $\Delta < 0,\ Y_m(t-\tau) = 1/\Omega_m \sin \Omega_m(t-\tau)$

$$U_{imn}^{v}(t) = \int_0^t U_{imn}^{T}(\tau)\, Y_m(t-\tau) \mathrm{e}^{-\zeta_m(t-\tau)}\, \mathrm{d}\tau$$

$$= \frac{1}{M_1}[A_1 \cos 2i\varphi t + A_2 \sin 2i\varphi t] + \frac{(2i\varphi)^2 + \gamma_{mn}^2}{M_1 M_2}\left[A_3 \cos\Omega_m t + A_4\frac{\sin \Omega_m t}{\Omega_m}\right]e^{-\zeta_m t} - \frac{\gamma_{mn}}{M_2}e^{-\gamma_{mn} t}, \tag{A.1}$$

$$W_{imn}^v(t) = \int_0^t W_{imn}^T(\tau)Y_m(t-\tau)e^{-\zeta_m(t-\tau)}\,d\tau = \frac{1}{\bar{M}_1}[B_1 \cos(2i+1)\varphi t + B_2 \sin(2i+1)\varphi t] + \frac{(2i+1)\varphi}{\bar{M}_1 M_2}\left[B_3 \cos \Omega_m t + B_4 \frac{\sin \Omega_m t}{\Omega_m}\right]e^{-\zeta_m t} - \frac{(2i+1)\varphi}{M_2}e^{-\gamma_{mn} t}, \tag{A.2}$$

Case 2: $\Delta = 0$ $(\omega_m = \zeta_m)$, $Y_m(t-\tau) = t - \tau$

$$U_{imn}^v(t) = \frac{1}{M_1}[A_1 \cos 2i\varphi t + A_2 \sin 2i\varphi t] + \frac{(2i\varphi)^2 + \gamma_{mn}^2}{M_1 M_2}(A_3 + A_4 t)e^{-\zeta_m t} - \frac{\gamma_{mn}}{M_2}e^{-\gamma_{mn} t}, \tag{A.3}$$

$$W_{imn}^v(t) = \frac{1}{\bar{M}_1}[B_1 \cos(2i+1)\varphi t + B_2 \sin(2i+1)\varphi t] + \frac{(2i+1)\varphi}{\bar{M}_1 M_2}[B_3 + B_4 t]e^{-\zeta_m t} - \frac{(2i+1)\varphi}{M_2}e^{-\gamma_{mn} t}. \tag{A.4}$$

Case 3: $\Delta > 0$, $Y_m(t-\tau) = 1/\Omega_m \sinh \Omega_m(t-\tau)$

$$U_{imn}^v(t) = \frac{1}{M_1}[A_1 \cos 2i\varphi t + A_2 \sin 2i\varphi t] + \frac{(2i\varphi)^2 + \gamma_{mn}^2}{M_1 M_2}\left[A_3 \cosh \Omega_m t + A_4 \frac{\sinh \Omega_m t}{\Omega_m}\right]e^{-\zeta_m t} - \frac{\gamma_{mn}}{M_2}e^{-\gamma_{mn} t}, \tag{A.5}$$

$$W_{imn}^v(\mathrm{t}) = \frac{1}{\bar{M}_1}[B_1 \cos(2i+1)\varphi t + B_2 \sin(2i+1)\varphi t] + \frac{(2i+1)\varphi}{\bar{M}_1 M_2}\left[B_3 \cosh \Omega_m t + B_4 \frac{\sinh \Omega_m t}{\Omega_m}\right]e^{-\zeta_m t} - \frac{(2i+1)\varphi}{M_2}e^{-\gamma_{mn} t}. \tag{A.6}$$

where

$$A_1 = \gamma_{mn}\omega_m^2 - (\gamma_{mn} + 2\zeta_m)(2i\varphi)^2, \quad A_2 = 2i\varphi[\omega_m^2 + 2\gamma_{mn}\zeta_m - (2i\varphi)^2],$$

$$A_3 = \gamma_{mn}(2i\varphi)^2 - (\gamma_{mn} - 2\zeta_m)\omega_m^2, \quad A_4 = \omega_m^2[(2i\varphi)^2 - \omega_m^2 + \zeta_m(2\zeta_m - \gamma_{mn})] - (2i\varphi)^2\gamma_{mn}\zeta_m,$$

$$B_1 = (2i+1)\varphi[(2i+1)^2\varphi^2 - \omega_m^2 - 2\gamma_{mn}\zeta_m], \quad B_2 = \gamma_{mn}\omega_m^2 - (2i+1)^2\varphi^2(\gamma_{mn} + 2\zeta_m),$$

$$B_3 = [(2i+1)^2\varphi^2 + \gamma_{mn}^2][2\zeta_m(\gamma_{mn} - 2\zeta_m) - (2i+1)^2\varphi^2 + \omega_m^2],$$

$$B_4 = [(2i+1)^2\varphi^2 + \gamma_{mn}^2]\{(\gamma_{mn} - \zeta_m)[(2i+1)^2\varphi^2 + 2\zeta_m^2 - \omega_m^2] + 2\zeta_m(\omega_m^2 - \zeta_m^2)\},$$

$$M_1 = [(2i\varphi)^2 - \omega_m^2 + 2\zeta_m^2]^2 + 4(\omega_m^2 - \zeta_m^2)\zeta_m^2,$$

$$\bar{M}_1 = [(2i+1)^2\varphi^2 - \omega_m^2 + 2\zeta_m^2]^2 + 4(\omega_m^2 - \zeta_m^2)\zeta_m^2,$$

$$M_2 = (\zeta_m - \gamma_{mn})^2 + \omega_m^2 - \zeta_m^2.$$

References

[1] Y-Y. Yu, Thermally induced vibration and flutter of a flexible boom, Journal of Spacecraft and Rockets 6 (1969) 902–910.
[2] B.A. Boley, Thermally induced vibrations of beams, Journal of the Aeronautical Sciences 23 (2) (1956) 179–181.
[3] B.A. Boley, Approximate analyses of thermally induced vibrations of beams and plates, Journal of Applied Mechanics 39 (1972) 212–216.
[4] G.D. Manolis, D.E. Beskos, Thermally Induced vibrations of beam structures, Computer Methods in Applied Mechanics and Engineering 21 (1980) 337–355.
[5] J. Kidawa-Kukla, Vibration of a beam induced by harmonic motion of a heat source, Journal of Sound and Vibration 205 (1997) 213–222.
[6] J.V. Beck, K.D. Cole, A. Haji-Sheikh, B. Litkouhi, Heat Conduction Using Green's Functions, Hemisphere Publishing Corporation, London, 1992.
[7] N.S. Al-Huniti, M.A. Al-Nimr, M. Naij, Dynamic response of a rod due to a moving heat source under the hyperbolic heat conduction model, Journal of Sound and Vibration 242 (2001) 629–640.
[8] A.S. Carslaw, F.C. Jaeger, Conduction of Heat in Solids, Oxford University Press, Oxford, 1959.
[9] I.S. Gradstein, I.M. Ryzhik, Tables of Integrals Series and Products, Academic Press, New York, 1980.
[10] A.G. Butkovsky, Green's Functions and Transfer Functions Handbook, Ellis Horwood Limited, Halsted Press, New York, 1982.